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'A Coupled Theory of Electromagnetism and Gravitation'

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Abstract

A coupling of electromagnetism with a previously developed scalar theory of gravitation is presented. The principle features of this coupling are: (1) a slight alteration to the Maxwell equations, (2) the motion of a charged particle satisfies an equation with the Lorentz force appearing on the right side in place of zero, and (3) the energy density of the electromagnetic field appears in the gravitational field equation in a manner similar to the mass term in the Klein–Gordon equation. The field of a static, spherically symmetric charged particle is computed. The electromagnetic field gives rise to $1/r^2$ terms in the gravitational potential.

In a previous paper, Lindén (1972), a scalar theory of gravity was developed. In the present article I should like to indicate how this theory may be unified with electromagnetism and what the consequent changes to Maxwell's equations are. We shall adhere to the MKS system of units. Since the speed of light was taken to be related to the gravitational potential through the formula

$$c = c_0 e^{-2\Phi} \tag{1}$$

then we must expect that either the electric permittivity or the magnetic permeability on both will be dependent on the gravitational field. The electromagnetic field tensor, F_{ij} is derived from the vector potential A_i by the formula

$$F_{ij} = \partial_i A_j - \partial_j A_i \tag{2}$$

and is explicitly given by

$$F_{ij} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix} \tag{3}$$

In terms of contravariant components we have

$$F^{ij} = \mu \begin{pmatrix} 0 & -D_1 & -D_2 & -D_3 \\ D_1 & 0 & -H_3 & H_2 \\ D_2 & H_3 & 0 & -H_1 \\ D_3 & -H_2 & H_1 & 0 \end{pmatrix} \tag{4}$$

where we have used the relations

$$B = \mu H, \qquad D = \varepsilon E \qquad \text{and} \qquad 1 = c^2 \mu\varepsilon \tag{5}$$

Also, the metric tensor has the form

$$g_{ij} = \operatorname{diag}(c^2, -1, -1, -1) \tag{6}$$

in a Cartesian system of coordinates. We also define the dual to F_{ij}, F^{*ij}, as

$$F^{*ij} = \tfrac{1}{2}\varepsilon^{ijkl} F_{kl} \tag{7}$$

where ε^{ijkl} is the Levi–Cevita symbol. The dual reads explicitly.

$$F^{*ij} = \begin{pmatrix} 0 & B_1 & B_2 & B_3 \\ -B_1 & 0 & -E_3 & E_2 \\ -B_2 & E_3 & 0 & -E_1 \\ -B_3 & -E_2 & E_1 & 0 \end{pmatrix} \tag{8}$$

The Maxwell equations are

$$\partial_i(\sqrt{(-g)}F^{ij}) = \sqrt{(-g)}J^j \tag{9}$$

$$\partial_i(\sqrt{(-g)}F^{*ij}) = 0 \tag{10}$$

where J^j is the current density vector. In vector notation these equations read,

$$\nabla.(c\mu\mathbf{D}) = c\mu\rho \tag{11}$$

$$\nabla \times (c\mu\mathbf{H}) - \frac{\partial(c\mu\mathbf{D})}{\partial t} = c\mu\mathbf{J} \tag{12}$$

$$\nabla.(c\mathbf{B}) = 0 \tag{13}$$

$$\nabla \times (c\mathbf{E}) + \frac{\partial(c\mathbf{B})}{\partial t} = 0 \tag{14}$$

If we consider a system of N charged particles, the equations of motion and the field equations are derivable from a variational principle

$$\delta \int d^4x \mathscr{L}^{\#} = 0 \tag{15}$$

where the Lagrangean density is given by

$$\mathscr{L}^{e} = \tfrac{1}{4}\sqrt{(-g)}\,F_{ij}F^{ij} + \sum_{\alpha=1}^{N} e_{\alpha} \int ds\, c\mu\sigma_{\alpha}(x^{i}, z^{i})\, \dot{z}^{i}_{(\alpha)} A_{i} \tag{16}$$

where e_{α} is the charge of the αth particle, σ_{α} is the normalised charge distribution times $\sqrt{-g}$ and z^{i} its coordinates. This definition of σ is chosen by the analogy with a point source in four dimensions which is given by a four-dimensional delta function (i.e. a product of four simple delta functions) divided by $\sqrt{-g}$. Before leaving these preliminary considerations, we note that the first term in the free electromagnetic field Lagrangean density is

$$\tfrac{1}{2}Jc\left(B^{2} - \frac{1}{c^{2}}E^{2}\right) \tag{17}$$

where $J = h_1 h_2 h_3$ and the h_i are the scale factors in a coordinate system in which the metric tensor is

$$g_{ij} = \operatorname{diag}(c^{2}, -h_1^{2}, -h_2^{2}, -h_3^{2}) \tag{18}$$

The Lagrangean density for the gravitational field and a system of N particles was shown in the previous paper to be

$$\mathscr{L}^{G} = \tfrac{1}{2}\sqrt{(-g)}\,\Phi^{i}\Phi_{i} + \frac{4\pi}{2}\sum_{\alpha=1}^{N} \frac{m_{\alpha} G}{c_0^{2}} \int ds\,\rho_{\alpha}(x^{i}, z^{i})(1+\Phi)^{2} g_{ij}\dot{z}^{i}\dot{z}^{j} \tag{19}$$

The Lagrangean density for the combined gravitational field, the electromagnetic field and a system of charged particles is then

$$\mathscr{L} = \mathscr{L}^{G} + \kappa\mathscr{L}^{e} \tag{20}$$

where κ is the constant, coupling the electromagnetic to the gravitational field.

The variation of the action

$$\mathscr{A} = \int d^{4}x\,\mathscr{L} \tag{21}$$

with respect to the z^{i} gives the equations of motion. In doing so we shall take ρ_{α} and σ_{α} to be delta functions which gives

$$\frac{d}{ds}\left[\frac{4\pi m_{\alpha} G(1+\Phi)^{2}}{c_0^{2}} g_{ij}\dot{z}^{j} + \kappa e_{\alpha}\mu c A_{i}\right] = \frac{\partial}{\partial z^{i}}\left[\frac{2\pi m_{\alpha} G(1+\Phi)^{2}}{c_0^{2}} g_{jk}\dot{z}^{j}\dot{z}^{k} + \kappa e_{\alpha}\mu c\dot{z}^{l} A_{l}\right] \tag{22}$$

where we have dropped the label α on z since it is clear this is the equation of motion of the αth particle.

To evaluate the coupling constant, we set the gravitational field to zero and introduce $z^0 = t$ as the independent variable. Then we have that c becomes c_0 and

$$\frac{d}{dt}\left[\frac{m_\alpha G}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}\frac{dz_{i(\alpha)}}{dt}+\frac{\kappa}{4\pi}e_\alpha \mu_0 c_0^4 A_i\right] = +\frac{\kappa}{4\pi}e_\alpha \mu_0 c_0^4 \frac{dz^k_{(\alpha)}}{dt}\partial_i A_k \quad (23)$$

We see that

$$\kappa = \frac{4\pi G}{\mu_0 c_0^4} = \frac{4\pi \varepsilon_0 G}{c_0^2} \quad (24)$$

and thus κ is about 10^{-36} Farad/kg. Substituting from (24) into (22) gives

$$\frac{d}{ds}[m_\alpha(1+\Phi)^2 g_{ij}\dot{z}^j - e_\alpha \varepsilon_0 \mu c A_i] = \frac{\partial}{\partial z^i}[\tfrac{1}{2}m_\alpha(1+\Phi)^2 g_{jk}\dot{z}^k - e_\alpha \varepsilon_0 \mu c A_j]\dot{z}^j \quad (25)$$

The second term in the brackets on the left side of (25) may be written

$$\frac{\partial}{\partial z^j}(\mu c A_i)\dot{z}^j \quad (26)$$

so that (25) becomes

$$\frac{d}{ds}[m_\alpha(1+\Phi)^2 g_{ij}\dot{z}^j] - \frac{\partial}{\partial z^i}\left[\frac{m_\alpha}{2}(1+\Phi)^2 g_{jk}\right]\dot{z}^j\dot{z}^k = e_\alpha \varepsilon_0[\partial_j(\mu c A_i) - \partial_i(\mu c A_j)]\dot{z}^j \quad (27)$$

It will be convenient to introduce the dependence of the electromagnetic influence parameters, ε and μ on Φ. A sufficient choice according to equations (1) and (5) is†

$$\mu = \mu_0 e^{2\Phi}$$
$$\varepsilon = \varepsilon_0 e^{2\Phi}$$

so that (27) reduces to

$$\frac{d}{ds}[(1+\Phi)^2 g_{ij}\dot{z}^j] - \frac{1}{2}\frac{\partial}{\partial z^i}[(1+\Phi)^2 g_{jk}]\dot{z}^j\dot{z}^k = \frac{e_\alpha}{m_\alpha c_0}\dot{z}^j F_{ji} \quad (28)$$

The right side of (28) is simply the Lorentz force acting on the particle.

The action functional, $\mathscr{A}$, is dependent on the gravitational potential Φ, the electromagnetic potentials A_i and the coordinates $Z^i_{(\alpha)}$ of the N particles, $\alpha = 1,\dots N$. We might express this dependence by the formula

$$\mathscr{A} = \mathscr{A}[\Phi, A_i, Z^i_{(1)}, Z^i_{(2)}, \dots Z^i_{(N)}] \quad (29)$$

† Prof. Poeverlein has remarked to me that this choice has the advantage that the wave impedance of an electromagnetic wave is independent of Φ. This is a good basis for making this choice.

One has all in all N equations of motions such as (28) by varying the Z^i. By varying the A_i the Maxwell equations (9) and (10) result and finally by varying Φ we have the equation of the gravitational field.

Now, the only term in the electromagnetic Lagrangean, $\mathscr{L}^{\mathscr{E}}$, which contains the gravitational potential, Φ, is the first term, which is written in terms of B and E field in line (17) and this term depends only on Φ and not its gradient. Thus, the field equation follows from the Euler–Lagrange equation which is

$$\frac{\delta}{\delta x^i}\left(\frac{\partial \mathscr{L}^G}{\partial \Phi_i}\right)-\frac{\partial \mathscr{L}^G}{\partial \Phi}=\kappa\frac{\partial \mathscr{L}^{\mathscr{E}}}{\partial \Phi} \tag{30}$$

The left-hand member of (30) gives rise to the field equation for the gravitational field alone and has already been derived in the previous paper. Thus, (30) then becomes

$$\frac{\partial}{\partial x^i}\left(\sqrt{(-g)}g^{ij}\frac{\partial \Phi}{\partial x^j}\right)-\frac{1}{2}\frac{\partial\sqrt{(-g)}g^{ij}}{\partial \Phi}\Phi_i\Phi_j$$
$$-\frac{4\pi G}{2c_0^2}\sum_{\alpha=1}^{N} m_\alpha \int ds\,\delta^4(x^l-z^l_{(\alpha)})\dot z^i_{(\alpha)}\dot z^j_{(\alpha)}\frac{\partial}{\partial \Phi}[(1+\Phi)^2 g_{ij}]$$
$$=-\frac{4\pi\varepsilon_0 G}{c_0^2}\tfrac{1}{4}F_{ij}F_{mn}\frac{\partial}{\partial \Phi}(\sqrt{(-g)}g^{im}g^{jn}) \tag{31}$$

or in vector notation for Cartesian coordinates (performing the integration using $t=z^0$ as the integration variable).

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial e^{-\Phi}}{\partial t}\right)-\nabla^2\Phi$$
$$+\frac{4\pi e^{-\Phi}}{c_0^2}\sum_{\alpha=1}^{N} m_\alpha G\,\delta^3(x-z(t))\sqrt{\left(1-\frac{v_\alpha^2}{c^2}\right)}\left(1+\Phi-\frac{2(1+\Phi)^2}{1-\frac{v_\alpha^2}{c^2}}\right)$$
$$=+\frac{1}{4}\frac{\kappa}{c_0}e^{\Phi}\frac{\partial}{\partial \Phi}\left(cB^2-\frac{1}{c}E^2\right) \tag{32}$$

If (31) is divided throughout by $\sqrt{-g}$ then the first term on the left is the wave operator, the second term is the energy density of the gravitational field times $4\pi G/c_0^2$, the third term is the energy-momentum density of the material sources of field times $4\pi G/c_0^2$ and the term on the right is the energy density of the electromagnetic field times $4\pi G/c_0^2$. The energy density of the electromagnetic field is known not to be an invariant. However, these interpretations suggest an invariant definition for the energy density of the electromagnetic field as

$$\mathscr{E}=-\frac{1}{2}\frac{\varepsilon_0}{\sqrt{-g}}F_{ij}F_{mn}\frac{\partial}{\partial \Phi}(-\sqrt{(-g)}g^{im}g^{jn}) \tag{33a}$$

or in the Cartesian system this reads

$$\mathscr{E} = -\frac{1}{2}\frac{\varepsilon_0}{c}\frac{\partial}{\partial\Phi}\left(cB^2 - \frac{1}{c}E^2\right) \qquad (33b)$$

the quantity in parentheses is recognised as one of the Maxwell invariants. That this indeed is the energy density is clear from the fact that $c = c_0 e^{-2\Phi}$ so that (33b) becomes

$$\mathscr{E} = \varepsilon_0\left(B^2 + \frac{1}{c^2}E^2\right)$$

(In conventional units this is twice the energy density divided by c^2.)

The same result is obtained in any coordinate system, since the action principal is made to undergo the transformation; so, for example, the result of a Lorentz transformation would be that (33b) would read

$$\mathscr{E} = -\frac{1}{2}\frac{\varepsilon_0}{c'}\frac{\partial}{\partial\Phi'}\left(c' B'^2 - \frac{1}{c'}E'^2\right)$$

which is seen to give the energy density in the new frame.

The third term on the left side of (31) which was referred to as the energy-momentum density of matter is recognised as being similar to the trace of the usual matter tensor. In fact, the differential of the quantity in brackets may be written as

$$2(1+\Phi)g_{ij} + (1+\Phi)^2\frac{\partial g_{ij}}{\partial\Phi}$$

For weak fields, g_{ij} may be expanded in a power series in Φ about the Minkowski metric η_{ij}. Thus

$$g_{ij} = \eta_{ij} + \Phi h_{ij}^{(1)} + \cdots$$

so that to terms linear in Φ these two terms may be written as

$$2(1+\Phi)g_{ij} + (1+\Phi)^2 h_{ij}^{(1)}$$

or

$$2(1+\Phi)\eta_{ij} + (1+4\Phi)h_{ij}^{(1)}$$

This term in the field equation (31) becomes

$$\frac{4\pi G}{c_0^2}\sum m_\alpha \int ds\,\delta^4(x - z_{(\alpha)}^i)\left[1 + \Phi + \tfrac{1}{2}(1+\Phi)^2 h_{ij}^{(1)}\dot{z}_{(\alpha)}^i\dot{z}_{(\alpha)}^j\right]$$

The deviation of the brackets from unity represents the influence of the gravitational field and the state of the motion on the active gravitational mass. We shall define the active gravitational mass (in units of length) of a body as

$$\sqrt{(-g)}\,M_\alpha^A = -\frac{Gm_\alpha}{2c_0^2}\int ds\,\delta^4(x^i - z_{(\alpha)}^i)\,\dot{z}_{(\alpha)}^i\dot{z}_{(\alpha)}^j\frac{\partial}{\partial\Phi}\left[(1+\Phi)^2 g_{ij}\right] \qquad (34a)$$

The inertial passive mass may by definition be read off from the equations of motion. Thus

$$\sqrt{(-g)}\, M_\alpha^P = \frac{m_\alpha G}{c_0^2}(1+\Phi)^2 \tag{34b}$$

Had the distribution of the particle not been chosen as $\delta^4(x^i - z^i_{(\alpha)})$ but rather some scalar density $\rho(x^i, z^i_{(\alpha)})$ then (34b) would read

$$\sqrt{(-g)}\, M_\alpha^P = \frac{m_\alpha G}{c_0^2}\int d^4 x \rho(x^i, z^i_{(\alpha)})(1+\Phi(x^i))^2 \tag{34c}$$

and thus the structure of matter would influence its motion. This would mean a violation of Eötvös experiments, i.e. a non-null result, in principle. It is obvious, however, that to test such a supposition would require bodies with the dimension of planets. These two definitions of mass have a most satisfying interpretation in terms of Aristotelian metaphysics. The quantity m_α is the inherent quality and quantity of matter. It is the indestructible form from which the attributes of active and passive mass derive. We may also define an equivalent electromagnetic mass-radius density

$$\sqrt{(-g)}\, M^\varepsilon = -\frac{1}{4}\frac{G}{c_0^2}\varepsilon_0 F_{ij} F_{mn}\frac{\partial}{\partial \Phi}(\sqrt{(-g)}\, g^{im} g^{jn}) \tag{35a}$$

Similarly, we may define an equivalent mass-radius density to the gravitational field

$$\sqrt{(-g)}\, M^G = -\frac{1}{8\pi}\frac{\partial \sqrt{(-g)}\, g^{ij}}{\partial \Phi}\Phi_i \Phi_j \tag{35b}$$

Thus, (31) may be written

$$\Box^2 \Phi = -4\pi \sum M_\alpha^A + 4\pi M^\varepsilon + 4\pi M^G \tag{36a}$$

where

$$\Box^2 = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}\left(\sqrt{(-g)}\, g^{ij}\frac{\partial}{\partial x^j}\right)$$

In Cartesian coordinates with the dependence of the speed of light from (1) introduced, (31) simplifies to the form

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial e^{-\Phi}}{\partial t}\right) - \nabla^2 e^{-\Phi} = 4\pi e^{-\Phi}\sum M_\alpha^A - 4\pi M^\varepsilon e^{-\Phi} \tag{36b}$$

where now

$$M_\alpha^A = -\frac{m_\alpha G}{c_0}\delta^3(x - z(t))\sqrt{(1-\beta_\alpha^2)}(1+\Phi)\left[1 - \frac{2(1+\Phi)}{1-\beta_\alpha^2}\right]$$

$$M^\varepsilon = \varepsilon_0\left(B^2 + \frac{1}{c^2}E^2\right)$$

and

$$\beta_\alpha = \frac{v_\alpha}{c}$$

For such velocities that $\beta_\alpha \ll 1$ then

$$M_\alpha^A = \frac{m_\alpha G}{c_0^2}(1+\Phi)(1+2\Phi)\delta^3(x - z_\alpha(t))$$

The 'electromagnetic mass' is thus seen to be repulsive. This interpretation is, however, quite crude since the energy density of the electromagnetic field is presumably not localised as a delta function. A better analogy for interpretation is to be found from wave mechanics. The electromagnetic mass appears as a potential well.

The electromagnetic field thus acts as a dispersive medium for gravitational waves. The wave equation for gravitational waves is, of course, non-linear; in fact, writing out the D'alembertian, (36b) reads

$$\frac{1}{\psi^2}\frac{\partial}{\partial t}\left(\frac{1}{\psi^2}\frac{\partial\psi}{\partial t}\right) - c_0^2\nabla^2\psi + 4\pi M^e\psi = -4\pi\sum_{\alpha=1}^{N} M_\alpha^A\,\delta(x - z_{(\alpha)}(t))\,\psi \qquad (37)$$

where we have set

$$\psi = e^{-\Phi}$$

It is not obvious that wave solutions do exist; however, to the extent that the equation may be linearised, i.e.

$$\psi \simeq 1 - \Phi$$

we obtain

$$-\frac{\partial^2\Phi}{\partial t^2} + c_0^2\nabla^2\Phi - 4\pi M^e\Phi - 4\pi\sum M_\alpha^A\,\delta(x - z_{(\alpha)}(t))\,\Phi$$
$$= -4\pi M^e - 4\pi\sum M_\alpha^A\,\delta(x - z_{(\alpha)}(t)) \qquad (38)$$

Field of a Static, Spherically Symmetric Charged Particle

For this case, the field equation (37) reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{de^{-\Phi}}{dr}\right) - 4\pi M^e e^{-\Phi} = -4\pi\frac{mG}{c_0^2}\frac{\delta(r)}{r^2} \qquad (39)$$

Let us denote the charge on the particle by Ze where e is the charge of an electron ($1\cdot6 \times 10^{-19}$ coul) and Z is the multiplicity of such charges. The electromagnetic mass density is given by

$$M^e = \frac{\varepsilon_0}{2c^2}E^2 = \frac{\varepsilon_0}{2c^2}\left(\frac{Ze}{4\pi\varepsilon r^2}\right)^2$$

The previously made choice as to how the electromagnetic influence parameters, μ and ε, should depend on Φ was

$$\varepsilon = \varepsilon_0 e^{2\Phi} \qquad \mu = \mu_0 e^{2\Phi} \tag{40}$$

The electromagnetic mass density then reads

$$M^e = \frac{\mu_0 Z^2 e^2}{2(4\pi)^2 r^2}$$

and is then independent of Φ.

The first two Maxwell equations, (11) and (12) simplify in that $c\mu = c_0\mu_0$, which is a constant. Also, the equations of motion, (28) simplify. The right-hand side reduces to the familiar expression for the Lorentz force.

Equation (39) may be written

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{de^{-\Phi}}{dr}\right) + \frac{\alpha}{r^4}e^{-\Phi} = -4\pi\frac{mG}{c_0^2}\frac{\delta(r)}{r^2} \tag{41}$$

where

$$\alpha = \frac{\mu_0 Z^2 e^2 G}{8\pi c_0^2}$$

which is in numerical terms approximately

$$\alpha = 10^{-72} m^2 \times Z^2$$

If we express the solution to (41) as

$$e^{-\Phi} = 1 + \frac{mG}{c_0^2 r} + f(r) \tag{42}$$

then $f(r)$ satisfies the equation

$$\frac{1}{r^2}\frac{d}{dr}(r^2 f') + \frac{\alpha}{r^4}\left(1 + \frac{mG}{c_0^2 r} + f\right) = 0 \tag{43}$$

We seek solutions which vanish at infinity. Thus, we assumed a series expansion of the form

$$f(r) = \sum_{n=1}^{\infty} \frac{a_n}{r^n}$$

Substituting into (43) and solving for the coefficients we find for the first three

$$a_1 = 0$$

$$a_2 = \tfrac{1}{8}\alpha$$

$$a_3 = -\frac{mG}{18c^2}\alpha$$

and generally the recursion formula

$$a_{n+2} = -\frac{\alpha}{2(n+2)^2} a_n$$

so that

$$e^{-\Phi} = 1 + \frac{mG}{c_0{}^2 r} - \frac{\mu_0 e^2 GZ^2}{64\pi c_0{}^2 r^2} + \cdots \tag{44}$$

If we apply these results to a proton we find that approximately

$$e^{-\Phi} = 1 + \frac{10^{-54}}{r} - \frac{10^{-73}}{r^2} \tag{45}$$

The quantity 10^{-54} is the gravitational radius of the proton. The two terms on the right are of the same magnitude at $r = 10^{-19}$ m. For the sun the gravitational radius is 1470 m. It is hard to imagine that the second term could be important for objects the size of stars. In fact, for a star the size of the sun in order that this term should be comparable with the $1/r$ term it would require 10^{38} free protons.

Summary

The coupling of the previously developed scalar theory of gravitation to the electromagnetic field has been accomplished. The coupling constant is determined to be about 10^{-38}. The speed of light and the electromagnetic influence parameters are given by the formulas $c = c_0 e^{-2\Phi}$, $\mu = \mu_0 e^{2\Phi}$ and $\varepsilon = \varepsilon_0 e^{2\Phi}$. The Maxwell equations read in vector notation

$$\nabla . \mathbf{D} = \rho$$

$$\nabla \times \mathbf{H} - \frac{\partial D}{\partial t} = J$$

$$\nabla . (c\mathbf{B}) = 0$$

$$\nabla \times (c\mathbf{E}) + \frac{\partial (c\mathbf{B})}{\partial t} = 0$$

So it is only the last two Maxwell equations which are altered by the gravitational field.

References

Lindén, T. L. J. (1972). *International Journal of Theoretical Physics*, Vol. 5, No. 5, p. 359.